# Japanese High School Text "Mathematics B"

## $N$ -th power root

In general, a non-zero complex number, $a = r(\cos\theta + i\sin\theta)$, has the following n complex numbers as $n$-th power roots.

$$z_n = \sqrt[n]{r}\left\{\cos\left(\frac{\theta}{n} + \frac{360^\circ}{n} \times k\right) + i\sin\left(\frac{\theta}{n} + \frac{360^\circ}{n} \times k\right)\right\}\ (k = 0,\ 1,\ 2,\ \cdots,\ n-1),$$

where $\sqrt[n]{r}$ is a positive $n$-th power root of a positive number r.

## An angle made by two vectors

Suppose two vectors $\overrightarrow{a} = (a_1, a_2)$ and $\overrightarrow{b} = (b_1, b_2)$ are non-zero vectors, $\theta$ is the angle made by these two vectors, and $0^\circ \leqq \theta \leqq 180^\circ$. Since $\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\cos\theta$,

$$\cos\theta = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|} = \frac{a_1 b_1 + a_2 b_2}{\sqrt{{a_1}^2 + {a_2}^2}\sqrt{{b_1}^2 + {b_2}^2}}$$

## A point that divides a segment into $m$:$n$

Suppose two points, A($\overrightarrow{a}$) and B($\overrightarrow{b}$), are not identical, $m + n \neq 0$, and a point, P($\overrightarrow{p}$), divides a segment AB into $m : n$. Then,

$$\overrightarrow{p} = \frac{n\overrightarrow{a} + m\overrightarrow{b}}{n + m}$$

Particularly, when the midpoint of a segment AB is M($\overrightarrow{m}$),

$$\overrightarrow{m} = \frac{\overrightarrow{a} + \overrightarrow{b}}{2}$$

## Probability distribution

Suppose a random variable $X$ can take the following n values $x_1$, $x_2$, ......, $x_n$, and the probability of an event $X = x_i$ is $p_i$. Then,

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|---|---|
| Mean | $m = E(X) = \sum_{i=1}^{n} x_i p_i$ |
| Variance | $V(X) = E((X - m)^2) = \sum_{i=1}^{n} (x_i - m)^2 p_i$ |
| | $V(X) = E(X^2) - m^2 = \sum_{i=1}^{n} x_i^2 p_i - m^2$ |
| Standard deviation | $\sigma(X) = \sqrt{V(X)}$ |